

# Computational Issues in Oil Field Applications

## Abstract

This report provides a summary by participants of the long program on Computational Issues in Oil Field Applications at the Institute for Pure and Applied Mathematics (IPAM) at the University of California Los Angeles (UCLA), March 20 – June 9, 2017. It provides brief overviews of the three one-week workshops, discusses some nuclei of cooperation between participants as developed during the long program, and highlights some further work performed by participants while resident at IPAM.

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## Introduction

The world is increasingly reliant on unconventional (e.g., shale gas, heavy oil) and deep offshore resources that are difficult and expensive to find and produce. The computational challenges associated with these exploration and production operations are substantial. Specific issues include reliably imaging and characterizing deep subsurface oil and gas reservoirs, accurately simulating flow through these highly heterogeneous systems, and applying these modeling capabilities to quantify uncertainty and optimize field performance. Complications arise from the multiphysics and multiscale character of the wave propagation and fluid flow problems, from the need to perform data assimilation for different properties over a range of scales, and as a result of the challenging model-based optimization problems associated with maximizing reservoir performance.

This paper addresses three main areas of attention, in line with the three workshops that formed cornerstones of the program, and thereafter addresses various “nuclei of cooperation” that were formed during the long program.

## Workshop summaries

### Workshop 1: Multiphysics, Multiscale, and Coupled Problems in Subsurface Flow

Mary Wheeler (University of Texas at Austin)

Excellent overviews of numerical homogenization and multiscale methods for flow problems were presented. Also industrial perspective of multi-physics and coupled problems involving compositional multiphase flow systems, poro-mechanics, near wellbore stability, and surface facilities was discussed. The multidisciplinary workshop talks provided opportunity for attendees to obtain a broader perspective required in modeling field problems. Namely questions of uncertainty, numerical accuracy and the importance of treating multiple processes were considered.

### Workshop 2: Full Waveform Inversion and Velocity Analysis

Yunan Yang (University of Texas at Austin), William W. Symes (Rice University) and Maarten de Hoop (Rice University)

Seismic data contains interpretable information about subsurface properties. Imaging predicts the spatial locations as well as specifies the size of Earth properties that are useful in exploration seismology. The inverse method in the imaging predicts more physical properties if a full wave equation is employed instead of an asymptotic far-field approximation to it. This so called full waveform inversion (FWI) is a data-driven method to obtain high resolution subsurface properties by minimizing the difference between observed and synthetic seismic waveforms.

The notion of FWI was first brought up three decades ago, and has been actively studied as computing power has increased. Currently FWI can achieve stunning clarity and resolution. Both academia and industry have been actively working on the algorithms, software and workflows. However, this technique is facing many challenges. Research on major difficulties were covered in this workshop.

First, the physics of seismic waves are complex and we need more accurate forward modeling in inversion going from pure acoustic waves to anisotropic viscoelasticity. Recent developments focus on the multiscale, multiparameter and multi-mode modeling. Corresponding numerical techniques are studied in order to deal with the nonlinearity in this more complicated but more accurate forward modeling.

Second, it is well known that FWI produces incorrect results due to lack of low frequencies, data noise and inadequate starting model. This is mainly due to the ill-posedness of the inverse problem which we treat as a PDE-constrained optimization. To tackle this problem, the research directions can be grouped into two main ideas. One idea is to replace the popular least-squares norm with other objective functions in optimization for wider basin of attraction. The other idea is to hugely expand the dimensionality of the unknown model by adding non-physical coefficients. The additional coefficients convexify the problem and fit the data better.

The third topic covered in the workshop is about the uncertainty estimation and quantitative outputs of FWI. It has crucial importance for the industry in terms of risk management and dry well issues as well as geological and geodynamic interpretation of images. There are research ideas from the Data Assimilation community and the ensemble methods are also borrowed here, which are the main topics of Workshop 3: *Data Assimilation, Uncertainty Reduction, and Optimization for Subsurface Flow*.

Current challenges in oil and gas exploration and field development require the use of all the available information for enhancing resolution and quantification of the resources. The fourth topic is about incorporating non-seismic data and constraints in FWI such as passive and active source EM and gravity. Joint inversion methods have been developed and applied to address this problem and approach the global minimum of the solution through data integration.

The fifth topic in the workshop is about source inversion. It is extremely meaningful in earthquakes where we search for kinematic sources and in reservoir simulation where people care about microseismic sources to understand the flow and deformation.

To sum up, FWI is a challenging data-fitting procedure with several difficulties, but it is also quite attractive and promising since the technique extracts very high resolution images of the subsurface properties. The workshop gathered both expert and developing researchers from mathematics and geophysics and stimulated lively discussions of methodological weaknesses and underlying physical and mathematical challenges. Future developments will require further collaboration between the two communities.

### **Workshop 3: Data Assimilation, Uncertainty Reduction, and Optimization for Subsurface Flow**

Lou Durlowsky (Stanford University)

Workshop 3 addressed a range of topics in data assimilation, uncertainty reduction, and optimization in reservoir simulation and related flow problems. Data assimilation and uncertainty reduction must be performed because reservoir production data (e.g., well injection and production rates) invariably differ from simulation predictions. It is thus necessary to modify the reservoir model, typically comprised of permeability and porosity values in all cells in a grid, such that models sampled from the posterior distribution provide flow predictions that are in essential agreement with production (and any other) data. A range of talks on this topic were presented. Ensemble methods such as ensemble Kalman filters (EnKF), ensemble smoothers (ES), and ensemble smoothers with multiple data assimilation (ES-MDA) have become very popular, and several talks described the properties and performance, as well as new developments, with these methods. The consensus seems to be that ES-MDA appears to be the current method of choice, though there are still challenges with this approach including ensemble collapse, the lack of reference results on posterior uncertainty, and the treatment of geologically complex (non-Gaussian) models. The use of pattern-based calibration using training images was discussed as a means of incorporating complex geological features into history matched models generated using ensemble methods. Particle methods, which attempt to focus the generation of posterior models in particular areas of the search space, were also described. These approaches have some positive features, though there have been very few applications of particle methods for realistic reservoir simulation problems.

Talks on related topics were also presented. These included work on developing an optimal waterflood pilot design, a project that entailed history matching with production and electromagnetic data, and methods to quantify the value of information in the context of reservoir production problems. The impact of numerical approximations and model error on uncertainty quantification was also discussed. A multiscale procedure for history matching naturally fractured reservoirs was introduced. This approach is promising in that terms appearing directly in the coarse-scale flow matrix are determined during the data assimilation, which avoids the need to generate (and then compute multiscale bases for) different fracture realizations at each iteration of the data assimilation algorithm.

A number of talks targeted reservoir production optimization. Methods for the bi-objective optimization of short and long-term production, for optimizing using an ensemble of models and stochastic gradients, and for minimizing a measure of risk in geological CO<sub>2</sub> storage operations were presented. The latter work included the optimization of CO<sub>2</sub> injection well locations and time-varying injection rates, as well as the use of a reduced-order modeling procedure for well control optimization. Adjoint-based optimization was discussed in several talks – both in a reservoir simulation context and for a range of complex problems in geosciences. Optimization under uncertainty was addressed by several presenters. These talks included an assessment of treatments for handling oil price uncertainty (such as the use of conditional value at risk, CVaR), optimization following history matching with the goals of either maximizing or minimizing net present value (to provide upper and lower bounds on economic performance), and the determination of optimal

parameters and settings for multisegment wells including nozzle-type valves and autonomous inflow control devices.

Industry speakers discussed the application of optimization in the context of field studies. They emphasized the need for fast and robust methods that can be implemented quickly, the use of decision-tree-type approaches for optimization under uncertainty, and the optimization of steamfloods in mature fields. The latter application included a combination of data science approaches, approximate flow modeling, ensemble-based data assimilation, and optimization. A procedure for analyzing and optimizing data acquisition programs and pilot projects was also described. The goal in that work is to collect data such that the reduction in uncertainty is maximized.

## **Nuclei of Cooperation**

### **Local multiscale model reduction**

Eric Chung (Chinese University of Hong Kong), and Yalchin Efendiev and Wing Tat Leung (Texas A&M University)

Many practical applications, especially those considered in this IPAM Program, contain multiple scales and high contrast. These applications include flows in fractured media, processes in channelized porous media and other multi-physics phenomena. Due to scale disparity and the contrast, some type of local model reductions are used to solve these problems. In the numerical modelling and simulations of multiscale problems, it is difficult to construct local coarse scale models that give accurate solutions independent of scales and contrast.

During the IPAM program, we have developed a number of local multiscale model reduction techniques for a class of elliptic multiscale problems. Our approaches are based on the Generalized Multiscale Finite Element Method (GMsFEM). In particular, we construct multiscale basis functions in each coarse element via local spectral problems. The goal is to identify high-contrast features that need to be represented individually. These non-local features are typically channels and need separate basis functions. We emphasize that the localizations of channels are not possible, in general, and this is the reason for constructing basis functions for channels separately as discussed.

It was shown in our earlier works that the GMsFEM's convergence depends on the eigenvalue decay. However, it is difficult to show a coarse mesh dependent convergence without using oversampling and many basis functions. During the workshop, we came up a new idea and show a mesh-dependent convergence result with a minimal number of basis functions. This work is the first result that can be applied to high contrast media. The convergence analysis of the GMsFEM suggests that one needs to include eigenvectors corresponding to small eigenvalues in the local spectral decomposition. We note that these small eigenvalues represent the channelized features, as we discussed above. For high-contrast problems, the local solutions do not decay in channels and thus, we need approaches that can take into account the information in the channels when constructing the decaying local solutions. Our construction also leads to a class of domain decomposition preconditioners that give condition number almost equals one.

The above works result in two papers and one proceeding paper, that are under review. The works also lead to some new directions in model reduction for convection-diffusion, transport and wave problems in heterogeneous media.

### **Scattering control series for the acoustic wave equation**

Alexander Mamonov (University of Houston) and Maarten de Hoop (Rice University)

During the workshop on "Full Waveform Inversion and Velocity Analysis" M. de Hoop presented the work on the scattering control series for the acoustic wave equation and A.V. Mamonov presented results on using data-driven reduced order models (ROM) to preprocess acoustic data and image the sound speed. On the surface the two methods are quite distinct. While the scattering control series relies on the ideas of boundary control, ROM-based approach makes heavy use of the tools of linear algebra. However, certain properties of both methods imply that a deeper connection exists between the two. In particular, both methods rely on expressing the inner products of the wavefields in the bulk via the boundary data. Ultimately, both approaches use these relations to construct the localized wavefields to probe the unknown medium. As a consequence, both methods effectively suppress the multiple reflections. Also, probing the medium with localized wavefields allows to

obtain reconstructions of the sound speed. The authors find these similarities fascinating and they plan to collaborate to uncover the exact relation between the two methods.

### **Comparison of Various Parameterizations for Complex Geomodels**

Yimin Liu and Louis Durlofsky (Stanford University), and Jan-Dirk Jansen (Delft University of Technology)

Parameterized representations of permeability and porosity fields are very useful in the context of data assimilation, as they can act to retain the spatial correlation structure inherent in prior geomodels while reducing the number of variables that must be determined during history matching. Spatial correlation structure is typically defined either in terms of two-point statistics (Gaussian models) or training images (multipoint statistics). For Gaussian models, parameterizations based on Karhunen-Loève (K-L) expansions or Principal Component Analysis (PCA) have been shown to perform well. For models characterized by multipoint statistics, however, these approaches are inadequate, as they do not capture non-Gaussian effects (such as bimodality) present in, e.g., channelized geological systems. Alternate approaches based on PCA have been suggested by the Stanford group, including kernel PCA (KPCA) and regularized or optimization-based PCA (O-PCA). A recent approach using tensor-based reparameterization (originally developed within the context of image compression) has been studied by the Delft group. In joint work initiated at IPAM, we are comparing the various approaches on a consistent set of flow problems. We also plan to compute quantitative measures of the higher-order spatial statistics captured by the various methods. This work should enable us to clearly identify the relative advantages of these different parameterization procedures.

### **Discontinuous Galerkin Methods for the Modified Buckley-Leverett Equation**

Ying Wang (University of Oklahoma) and Yulong Xing (University of California Riverside)

The classical Buckley-Leverett (BL) equation is a simple model for two-phase fluid flow in a porous medium. One application is secondary recovery by water-drive in oil reservoir simulation. The modified BL (MBL) equation differs from the classical BL equation by including a balanced diffusive-dispersive combination. The dispersive term is a third order mixed derivatives term, which models the dynamic effects in the pressure difference between the two phases. The classical BL equation gives a monotone water saturation profile for any Riemann problem; on the contrast, when the dispersive parameter is large enough, the MBL equation delivers a non-monotone water saturation profile for certain Riemann problems as suggested by the experimental observations. As the equation contains both high order derivative terms and the solution may be discontinuous, we propose to develop high order discontinuous Galerkin (DG) method to solve such model numerically. The DG method is a class of finite element approximations using discontinuous, piecewise polynomials as both the solution and test-function spaces, which combines advantages of both finite element and finite volume methods, including high order accuracy, high parallel efficiency, flexibility for hp-adaptivity and straightforward implementation on arbitrary meshes in geometries. We plan to carry out rigorous analysis and numerical simulation to better understand the MBL model and its applications in oil reservoir simulation.

### **Estimation of Initial Fluid Distribution in Oil Reservoirs via Assimilation of Production Data**

Emilio Sousa (The University of Tulsa) and Rafael Moraes (Delft University of Technology)

The initial fluid contacts (specifically oil/water contact and gas/oil contact) are often uncertain. Traditionally, this information comes from well logs, which are spatially sparse and include different measurement errors. This information is of utmost importance in the predictive power of reservoir simulation models.

One alternative to reduce the uncertainty associated with this information is to consider the initial fluid contact as an uncertainty parameter in data assimilation (history matching) exercises. In the past few decades, due to different motivations, ensemble-based methods for data assimilation have increased in popularity among the research community. However, this class of methods fails to properly estimate parameters associated with the fluid model initialization. This is because it is not possible to maintain the statistical representation of the model after successive updates. Moreover, it could be of interest to estimate the fluid distribution in a given moment in time after some production has already occurred.

In this context, we consider gradient-based methods, which have not been widely used due to the difficulty of computing the adjoint equations, as an alternative. These methods can be used to robustly and accurately estimate the initial fluid distribution via the update of the uncertainty parameters associated with the initialization model.

### **Geostatistical Aspects of Simulation Algorithms Relevant to Uncertainty Quantification**

Jan Dirk Jansen (Delft University of Technology) and Asbjørn Riseth (Oxford University)

This research concerns aspects of simulation algorithms for porous media relevant to uncertainty quantification. It addresses the effect of initial parameter distributions of key uncertain parameters (e.g. permeabilities), and the consequences of their subsequent treatment in discretization approaches and solution methods, on the statistical properties of the solution. Initial results indicate that noticeable alterations of the probability density functions of the permeability field may result, in particular significant variance reduction, from standard algorithmic steps like harmonic averaging and transmissibility matrix assembly.

### **Further work performed while at IPAM**

#### **Multiscale Modelling of Phase Transformation**

Henry Martin (National Institute for Mathematical Science- Ghana, and Kwame Nkrumah University of Science and Technology (KNUST)

Attending the Conference I got updated with several techniques and more importantly it paved a way for me as the IPAM Director, Prof. Russ Caflisch, connected me to a Professor at UCLA after discussing with him my motivation and the techniques I would like to use for my PhD research. Currently, the Professor is one of my PhD Advisors and we are working on the atomistic stage (ab initio - DFT Calculation).

#### **Outcrop fracture extraction and fracture network characterization from three-dimensional point clouds**

Xin Wang (School of Earth Sciences, Zhejiang University, China)

Conventional fracture data collection methods are usually implemented on planar surfaces (such as bedding surfaces) or assuming they are planar; these methods may introduce sampling errors on uneven outcrop surfaces. In addition, they require large amounts of time and labor; yet, they provide a relatively small set of data that cannot be considered representative of the study region. Terrestrial laser scanners are increasingly used for fracture surveys because they can efficiently acquire large area, high-resolution, three-dimensional (3D) point clouds from outcrops. However, extracting fractures and other planar surfaces from 3D outcrop point clouds is still a challenging task. No method has been reported that can be used to automatically extract the full extent of every individual fracture from a 3D outcrop point cloud. We propose a method using a region-growing approach to address this problem, and to perform outcrop fracture network characterization on suppositional planes cutting through digital outcrop models (DOMs). In this method, criteria based on the local surface normal and curvature of the point cloud are used to initiate and control the growth of the fracture region. The suppositional plane is the best fit plane of the outcrop surface, and the fracture trace map is extracted on the suppositional plane so that the fracture network can be further characterized. In tests using outcrop point cloud data, the proposed method identified and extracted the full extent of individual fractures with high accuracy. Compared with manually acquired field survey data, our method obtained better-quality fracture data, thereby demonstrating the high potential utility of the proposed method. The amount of sampling errors introduced by the conventional methods and avoided by the new method on 16 uneven outcrop surfaces with different roughness are estimated. The results show that the proposed method can greatly extend the types of outcrop surfaces for outcrop fracture characterization with the suppositional plane cutting through DOMs.

#### **From partial differential equations (PDEs) with random coefficients to parameterized PDEs**

Michael Griebel and Guanglian Li (Institut für Numerische Simulation, Universität Bonn)

This work is a continuation of Griebel and Li (2017), and we are mainly concerned with optimally balancing among the Quasi Monte-Carlo approximation error, the truncation and the numerical approximation errors within the K-L expansion. This problem arises from numerical approximation of differential equations with random coefficients when the parameterization strategy is a necessary

step to transform a high dimensional stochastic domain into a finite dimensional parameter space, whereas the obtained parameterized problems can be solved by stochastic Galerkin Finite Element methods. The random coefficient  $\kappa(x,\omega)$  we are interested in is log-normal distributed given  $x \in D$  with  $D$  being the physical domain, which has no guaranteed uniform ellipticity condition and the random domain  $\Omega$  is unbounded. These facts render the proposed problem extremely challenging. This type of random coefficient has wide applications in oil field problems. The outcome of this research may play a crucial role in quantifying the error incurred during this parameterization process and will result in an accurate and efficient parameterized problem, in terms of the number of sampling points  $N$ , the terms maintained in the K-L expansion  $M$ , and the discretization mesh size  $h$  in the physical domain, to numerically solve the eigenvalue problems during the K-L expansion.

Griebel, M. and Li, G., 2017: On the decay rate of the singular values of bivariate functions. Submitted to *SIAM J. Numer. Analysis*. Also available as INS preprint No. 1702.

### **Nonlinear solver acceleration for reservoir simulation**

Asbjørn Nilsen Riseth and Patrick Farrell (Oxford University)

Reservoir simulators generate many nonlinear equations that must be solved efficiently. This takes up the majority of simulation time, and is classically done with the iterative Newton's method. Occasionally Newton's method fails, for example due to too long simulation time steps, and the remedy in most simulator software is to shorten the time step. However, a simple acceleration scheme known as Nonlinear GMRES, preconditioned with Newton's method, can both be more robust, and reduce the number of iterations required to solve the nonlinear equations. Previous work with Nonlinear GMRES for reservoir simulation has shown a 10% reduction in number of required Newton iterations, at little extra computational cost per iteration. During the long programme, we have worked on implementing this method in the MATLAB Reservoir Simulator Toolbox by SINTEF. This will enable us to test NGMRES-acceleration on cases that much better represent realistic oil production cases used in industry than what was done previously. The previous work investigated gravity-segregation problems for three-phase porous media flow, where the standard Newton method already did well. There is therefore great potential for NGMRES in production cases, where it is more common to experience problems with the standard Newton method.

### **Waveform inversion**

Frank Natterer (University of Münster)

First, I know now the state of the art in full waveform inversion. I have worked on this problem mainly on its medical aspects, mostly in mammography. The workshop showed me the state of the art in seismics and geophysical imaging. Since I know now that the problem of missing low frequencies is still open, I intensified my recent attempts to solve the problem by analytic continuation. Second, I got a lot of input to the FWD (falling weight deflectometer) problem. This problem has found much attention in the civil engineering community, but almost no attention in the inverse problems community. It is of relevance for road building companies. It is closely related to the missing low frequency problem, with additional emphasis on imaging materials with memory. I'm now doing numerical experiments for both problems, with software that is almost identical.

### **Multiscale Method for Transport Equation**

Rafael Moraes and Jan Dirk Jansen (Delft University of Technology)

A multiscale solution for the transport equation, in addition to a multiscale solution for the elliptic equation, could allow reservoir management studies (e.g. uncertainty quantification, data assimilation, and life-cycle optimization) to proceed more quickly and more accurately, with potential application to larger geological models.

The multiscale basis function construction for elliptic problems (with global behavior) has been a topic of extensive research. However, due to the nature of hyperbolic equations (local behavior), basis function construction is a challenge.

Although some works can be found in the literature, traditionally, fine scale resolution is used in order to capture the sharp saturation fronts. If coarser-scales are used in the transport equation solution, these sharp fronts are no longer represented and smoothed fronts are computed. This is due to numerical artifacts, which can lead to a non-physical representation of the flow in the porous media.

In the context of multiscale simulation, a challenge is to appropriately transfer the saturation information among the different solution scales.

In this work, we research an alternative to compute basis functions for hyperbolic equations based on streamlines traced from the flow equation solution in order to reduce the numerical artifacts of the solution.